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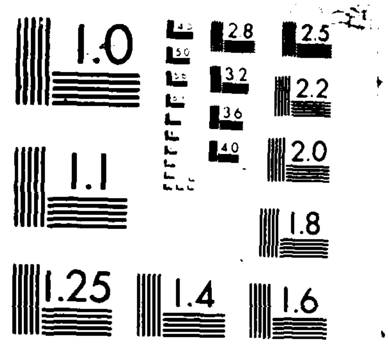

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THE EFFICIENCY, EFFECTIVENESS, AND EXCESS PRODUCTION CAPABILITY OF PRODUCTION UNITS

BY

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THE EFFICIENCY, EFFECTIVENESS, AND EXCESS PRODUCTION CAPABILITY OF PRODUCTION UNITS

BY

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September 1986

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FOREWORD

This report is the first of a series of two reports on the theory and application of techniques for measuring the Production Efficiency and Effectiveness of Production Units (PU). This report is intended to provide a theoretical development of the methodology for determining Production Efficiency and what is referred to herein as the "excess production capability", which is a measure of the amount by which a PU's outputs can decrease before the PU becomes inefficient. An equivalent way of looking at excess production capability is to think of it as "overproduction". With the material in this paper, the production output which a PU should achieve (as compared to other similarly producing units) can be determined. This paper deals with measuring the effectiveness of an arbitrary number of PU, with effectiveness being a measure of how well the PU does what it is told to produce (or missioned). A problem in measuring production effectiveness, which is a ratio of production output to mission, is that mission may not be based upon what a PU should be able to achieve. In this case, a PU may achieve a low effectiveness rating either because its mission is set too high or because its production is too low even though it can produce more with its existing resources (e.g., the PU is technically inefficient). If future missions are based upon past actual production, it may be seen that technically inefficient low producers will be given lower missions while technically efficient PU will be given higher missions. Thus, mission must somehow be based upon what the PU should be able to produce with normal (or typical) effort and that the methodology which determines what should have been produced must be able to recognize overproduction or underproduction of PU. The second paper in the series of two will use the principals explained in this paper and will apply the principals to the fifty-six US Army Recruiting Command Battalions.

DISCLAIMER

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ABSTRACT

Efficiency and Effectiveness are two measures commonly used to evaluate Production Units (PU). Whereas the Efficiency measurement shows how well a PU transforms its inputs into outputs and prescribes a corrective course of action to make an inefficient PU efficient relative to other similar PU, the Effectiveness measurement shows how well a PU is producing relative to a management defined mission or goal for production output. This report shows how Efficiency affects Effectiveness and presents a methodology for statistically comparing a PU's production to mission. A method for ranking both efficient and inefficient PU according to their "excess production capability" is introduced. This methodology also permits one to determine the overproduction of a PU, which is essential in the Effectiveness measurements. The material in this report will be applied to a subsequent separate report to the fifty-six US Army Recruiting Battalions.

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Per Mr. Juri Toomepuu, U. S. Army Recruiting Command

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THE EFFICIENCY, EFFECTIVENESS, AND EXCESS PRODUCTION CAPABILITY OF PRODUCTION UNITS

1. OVERVIEW OF EFFICIENCY AND EFFECTIVENESS.

To determine how well producing units are performing, two measures are frequently used. First, mission accomplishment or effectiveness is expressed as the ratio of a weighted sum of the various outputs to a weighted sum of the missions assigned for the outputs. In addition to determining appropriate weights, this measurement of mission effectiveness assumes that the mission has been properly determined so that an effectiveness ratio greater than one signifies an overproducer and a value of effectiveness less than one signifies an underproducer.

A second measure, somewhat similar to effectiveness, is technical efficiency (referred to as "efficiency" hereafter), which is defined as a ratio of a weighted sum of a producing unit's outputs to a weighted sum of the unit's inputs. As with effectiveness, the efficiency measurement requires the determination of appropriate weights. The material which follows shows how a set of weights can be determined using methodology pioneered by Charnes, Cooper, and Rhodes (CCR) [5] which gives each Production Unit (PU) the highest possible efficiency rating.

Whereas effectiveness answers the question of how well the PU did perform, efficiency answers the question of how well a PU should have performed given the set of input resources. This is an important distinction which must be made as illustrated by the followig example.

Assume that methodology exists to determine the following:

1. That a given PU's <u>mission</u> is 5 units.
2. That a given PU's <u>actual</u> production was 4 units.
3. That the PU's <u>possible</u> production is 6 units.

Measuring the effectiveness of the actual production, the PU is at 80% effectiveness and may be considered to be 20% overmissioned. However, the PU's possible effectiveness is 120%, meaning that with its potential production, it is 20% undermissioned.

Clearly, if a PU is consistently underproducing, it will appear to be overmissioned and the reason for reaching the wrong conclusion is that the PU is technically inefficient (e.g., can produce more than what it actually is). However, using the first calculation, we would reward technical inefficiency if management does, in fact,

reduce mission to coincide with actual production. This problem will be exacerbated further if future missions are determined as some function of the actual past production.

Yet, the effectiveness measurement must also be able to detect a PU which is "overproducing." Consider the following example.

Assume that a methodology exists to determine the following:

1. That a given PU's <u>mission</u> is 5 units.
2. That a given PU's <u>actual</u> production is 6 units.
3. That the PU <u>should</u> produce 4 units.

Measuring the effectiveness of the actual production, the PU is at 120% effectiveness and may be considered to be 20% under missioned. However, when calculating effectiveness based upon what the PU should produce, the PU is at 80% effectiveness, meaning that it is 20% overmissioned. If a PU is already producing above that which its contemporary PU are producing (e.g., is overproducing), then management would fail to properly reward the PU's achievement and would wrongly conclude that its mission could be increased by 20% when it should either be reduced by 20% or be given higher credit for overproduction.

For the PU which is overproducing consistently, it may appear to be undermissioned and the reason for reaching the wrong conclusion is that the PU has achieved a capacity to produce above that which other similar (but efficient) PU have achieved. This problem will persist and will be intensified if actual past production is used as a means of determining future mission.

In both of the examples above, the wrong conclusion was reached because the measure of effectiveness, based upon the PU's actual production, is the wrong measure to use. The measurement of effectiveness which is based upon actual production will tend to reward inefficiency with reduced future missions and will penalize overproduction by increased future mission. Thus, the measurement of effectiveness requires us to determine what a PU should produce if it is technically efficient. This production can be found, as will be shown, by determining the efficiency and then adjusting the actual production by the efficiency measurement. Then, effectiveness is found by comparing the efficiency adjusted production to mission.

Conversely, mission should be determined based upon what a technically efficient PU should be able to produce with given levels of input resources, when measured relative to other similarly producing units.

The material which follows describes the methodologies for first determining the efficiency adjustments to find what a PU should produce, and secondly, to determine the effectiveness of the PU and therefore, the changes in mission required for an equitable mission

for all PU.

2. TECHNICAL EFFICIENCY.

2.1 Introduction to Technical Efficiency Measurement.

To aid in illustrating how Technical Efficiency (TE) measurements are made, consider Figure 1.

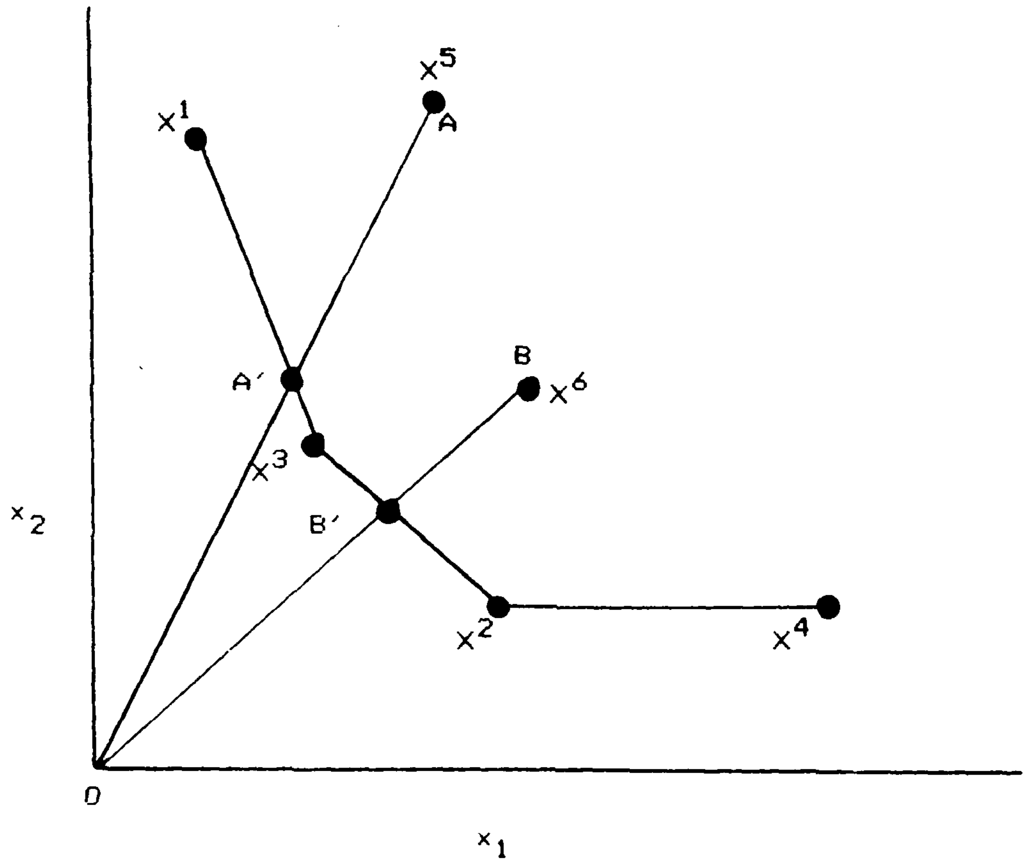

Figure 1. Efficiency Boundary Illustrated

Figure 1 illustrates six Production Units (PU), each having two inputs, x_1 and x_2, with each PU having a single output, y. Although each PU can have values of output which differ, if all inputs and outputs for a given PU are divided by the value of the PU′s single output value, each "normalized" PU has a single output equal to one in value and inputs with value equal to x_i^j/y^j, with j=1,...,6 and i=1 to 2 inputs. The normalized inputs for the PU are graphed in Figure 1, where all six PU have the same normalized output value equal to one. It should be noted, however, that the actual model which determines TE does not require normalization of data and

allows for an arbitrary number of inputs, outputs, and PU. This example was selected simply to illustrate in two dimensions what the TE model has the capability to do when PU have multiple inputs and outputs.

An objective in measuring TE is to find the smallest set of inputs which is capable of producing a given set of outputs for each individual PU separately. It can be seen in Figure 1 that X^5 clearly requires more inputs relative to the other PU to achieve the same unit value of output which the other PU achieve. That is, relative to X^1 and X^3, for example, X^5 requires more of both x_1 and x_2 inputs. Relative to X^2, however, X^5 requires less of x_1 than X^3 requires. The measurement of TE is a relative measurement, with the reference set of PU being the "standard" to which the inefficient PU are compared.

The complete set of reference PU are connected by the line through X^1, X^3, X^2, and X^4, and any PU which falls above or to the right of this frontier is inefficient with respect to some part of the frontier.

Clearly, X^5 can "move" to the frontier (become TE) if its inputs are reduced to point A′, for example. The ratio of the distance from the comparison point, A′, to the origin (A′O) and the distance from X^5 (A) to the origin (AO) is the value of Technical Efficiency (TE=A′O/AO), which is the amount by which each input must be decreased to bring X^5 (A) to the comparison point (A′) on the efficiency frontier (or boundary). Similarly, X^6 can be brought to the efficiency boundary by reducing its inputs (while holding its output constant) to bring it to a comparison point like B′. The ratio of the distance B′O and the distance BO is the value of the TE of X^6.

Considering X^4, it can be seen that it is on the efficiency boundary, yet it consumes more of x_1 input than X^2 consumes. Thus, even though the ratio of the distance from X^4 to the boundary is one, it consumes an excess amount of a resource, and this excess amount is referred to as a "slack" or underused resource.

For an arbitrary PU to be TE, then, it is necessary and sufficient for an inefficient PU to reduce its inputs by the value of the ratio measurement and by the value of the slack or underused capability. This brings the PU to an efficient comparison point on the efficiency boundary.

As shown in Figure 1, comparison points A′ and B′ are on the efficiency boundary and are considered to be in the set of "best" points in that no smaller inputs can produce the given PU's output. The comparison points, however, may not be actual producing

units, but could be composites of actual producing units which are TE. Hereafter, a comparison input point for an arbitrary PU will be labeled x'. By examining Figure 1, it should be clear that the efficiency boundary contains the set of all possible comparison points and that any given PU will have either one comparison point if it is TE or an infinite number of comparison points if it is not.

One problem, then, in measuring TE is to determine the comparison point for a specific PU. One method is to select a direction which "points" from the PU to the boundary and to follow that direction until no further input reductions can be achieved, providing the reduced input is capable of producing the PU's actual output. The direction which is selected will affect the comparison point, and hence, the TE.

In a manner similar to reducing inputs, TE can be measured by finding the largest set of outputs which a given PU's fixed inputs can produce. If a PU is TE, the set of output comparison points, Y', will contain only the PU's actual output. However, if a PU is not TE, the set of output comparison points will have an infinite number of comparison points. Hereafter, y' will denote a single comparison output point for an arbitrary PU. As with the discussion on inputs, a single comparison point, y', can be located by selecting a direction which "points" from the PU's actual output to the boundary of efficiency. If that direction is followed from the actual PU until no further increase in outputs is necessary for the given set of inputs, the comparison point will be located. Of course, having a mathematical model which makes the calculations needed to find the comparison points is needed. This model will be developed in the next section.

A third alternative for locating an efficient comparison point is to simultaneously select an input direction and output direction which "points" from a given PU to the respective efficiency boundary and to simultaneously follow the directions until no further input reductions or output increases can be made, providing that the reduced input, x', can produce the increased output, y'.

The model which determines a comparison point by only reducing inputs is called the Input Technology Model; the model which determines a comparison point by only increasing outputs is referred to as the Output Technology Model; and the model which allows for both input reduction and output increase to locate an efficient comparison point is referred to as a Mixed Technology Model. As will be shown in the next section, there is a symmetry in the mathematical requirements for measuring TE, resulting in a single model from which many other specific forms can be derived.

To illustrate this symmetry, section 2.2 will discuss the modeling considerations for determining the TE of any number of PU with an ar... ar... number of inputs and outputs. This will require the definition of certain concepts used in subsequent sections of this paper. Then, some of the modeling considerations will be illustrated using a special case of several PU, each having two inputs and a single output. Section 2.6 discusses the mathematical program which determines the TE of a PU. Repeated application of the program to each individual PU allows one to determine the TE of an arbitrary number of PU.

Chapter 3 discusses the methodology for determining the excess production capability and illustrates the method with data and with figures. Chapter 4 discusses Effectiveness measurement methodology.

2.2 Definitions.

Measuring Technical Efficiency has been shown in [10] to ... two-step process. First, a reference or a comparison point is determined for an individual PU using the set of all of the PU. Secondly, the TE for the PU is calculated as a function of the distance from the operating point of the PU to the comparison point. Repeated application of the two-step process for each of the PU determines their TE. In order to determine the comparison point to which an individual PU is to be compared, a mathematical model will be developed, which is similar to the model first developed in [5].

Consider $j=1,2,\ldots,n$ PU, with each PU producing an output vector $y^j=(y_1,y_2,\ldots,y_s)$ using an input vector, $x^j=(x_1,x_2,\ldots,x_m)$. Also, assume that each PU consumes the same types of resources to produce the sa... types of outputs, with the differences between PU being the quantities consumed or produced.

Define the point (x',y') to be the comparison point for the j-th PU. Also, assume that the point (x',y') can be found as some non-negative linear combination of the n PU. M, a matrix of outputs, and N, a matrix of inputs are defined:

$$M = \begin{bmatrix} y_1^1, y_2^1, y_3^1, \ldots, y_s^1 \\ y_1^2, y_2^2, y_3^2, \ldots, y_s^2 \\ \vdots \quad \vdots \quad \vdots \quad \quad \vdots \\ y_1^n, y_2^n, y_3^n, \ldots, y_s^n \end{bmatrix} \qquad N = \begin{bmatrix} x_1^1, x_2^1, x_3^1, \ldots x_m^1 \\ x_1^2, x_2^2, x_3^2, \ldots x_m^2 \\ \vdots \quad \vdots \quad \vdots \quad \quad \vdots \\ x_1^n, x_2^n, x_3^n, \ldots x_m^n \end{bmatrix}$$

As in [5], assume that each comparison point for the j-th PU will have no more input than the j-th PU and will produce at least as much output as the j-th PU. This means that (x',y') must satisfy:

The Production Technology Requirement.

$$x'_i \leq x^j_i \text{ for } i=1,2,\ldots,m \text{ and } y'_i \geq y^j_i \text{ for } i=1,2,\ldots,s \qquad (1)$$
$$\text{and } x' \Rightarrow\!> y' \text{ (} x' \text{ produces } y' \text{).}$$

The set (x',y') which is formed by the non-negative linear combinations of the n PU operating points is referred to as the Feasible Production Points. The "best" points in the set of Feasible Production Points are referred to as the Efficiency Boundary. If a PU is inefficient, it is inefficient because it consumes more inputs and/or produces less outputs than (x',y') on the Efficiency Boundary. For an inefficient point, the j-th PU inputs must "move" toward the comparison points on the Efficiency Boundary which are located by "looking" for points of lesser or equal inputs and greater or equal outputs.

No conditions are imposed on the measurement of the distance from the j-th DMU's operating point, (x^j,y^j), to (x',y') on the boundary. Define ρ' and ρ'' to be vectors and refer to the components of the vectors as component efficiencies. Now define:

ρ'_i, i=1,2,....,m:= input component efficiency such that

$$\rho'_i = x'_i / x^j_i$$

and (2)

ρ''_i, i=1,2,....,s:= output component efficiency such that

$$\rho'_i = y'_i / y^j_i .$$

Combining (1) and (2), it can be seen that $\rho'_i \leq 1$ and $\rho''_i \geq 1$. How TE is calculated from the component efficiencies will be discussed in detail in section 2.4. However, if the Technical Efficiency is is the average of the input component efficiencies, for example, then TE=1.0 (or 100%) if and only if all component efficiencies equal the value 1.0. Also, ρ'_i indicates the proportionate decrease in the i-th input for the PU to be efficient. The value ρ''_i indicates the proportionate increase in the i-th output for the PU to become TE. Finally, TE must be some function of the component efficiencies, ρ'_i and $1/\rho''_i$ to guarantee that an inefficient PU will have TE≤100%.

2.3 Relative Efficiency Example.

Using Figure 2, it will be shown that a comparison point can be found by following a "descent direction" toward the Efficiency Boundary and that there are many directions, and hence comparison points, for an inefficient PU. For ease of illustration, Figure 2 illustrates eight PU, each having two inputs and a single output. Each of the units' operating points have been normalized by dividing the value of the inputs and outputs by the value of the units' output. Thus, each PU in Figure 2 has unit output.

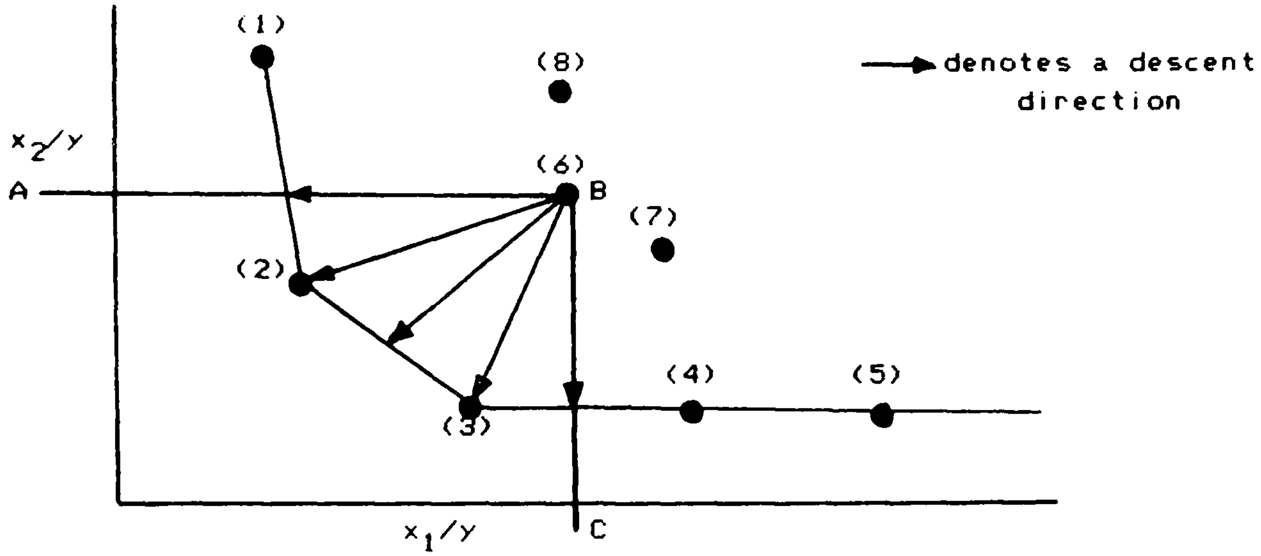

Figure 2. An Illustration of Relative Efficiency

In Figure 2, the Efficiency Boundary is the set of normalized inputs connecting PU (1), (2), (3), (4), and (5). These five PU and every convex combination of them constitutes a set of comparison points which are considered to be 100% efficient. Also, any point above and to the right of the boundary is inefficient relative to comparison points on the Efficiency Boundary. For PU (6), the points satisfying (1) are above the Efficiency Boundary and below the cone ABC.

Once a given PU is determined to be TE, it should also be clear that it is on the Efficiency Boundary and that if we were to look for points which are inefficient, we would look at the interior of the Efficiency Boundary. For inputs, the interior of the Efficiency Boundary is above and to the right of the set of efficient PU. For outputs, the interior of the Efficiency Boundary is below the set of efficient PU.

2.4 Step 1: Finding the Comparison Point.

Given some actual production point, (x^j, y^j), of the j-th PU, a comparison point, (x', y'), which satisfies (1) must be determined. The operating point x^j must be reduced by some $\xi^- \geq 0$ from x^j to x' in the direction $d^- \geq 0$, or

$x^j - \xi^- d^- \geq x'$, where $d^- \geq 0$ is a vector descent direction
and $\xi^- \geq 0$ is the distance which x^j is decreased along d^-. (3)

Similarly, we can specify that y^j must be increased along some ascent direction $d^+ \geq 0$ by some $\xi^+ \geq 0$, so that

$y^j + \xi^+ d^+ \leq y'$, where $d^+ \geq 0$ is a vector ascent direction
and $\xi^+ \geq 0$ is the distance which y^j is increased along d^+. (4)

By assumption, (x',y') is found as some non-negative linear combination of PU, so defining $z \geq 0$ to be a vector of multipliers,

$$x' = z^T N \quad \text{and} \tag{5}$$
$$y' = z^T M, \text{ where } z_i \geq 0 \text{ for } i=1,2,\ldots,n.$$

The basic model for finding the comparison point, (x',y'), is complete by noting that we wish to maximize the distance, ξ^-, for the inputs to be reduced and/or ξ^+, for the outputs to be increased, or

$$\text{Max } \xi^- + \xi^+$$

Subject To

$$x^j - \xi^- d^- \geq z^T N = x' \tag{6}$$
$$y' = z^T M \geq y^j + \xi^+ d^+$$
$$\xi^-,\ \xi^+,\ z \geq 0,\ d^- \geq 0 \text{ (some } d_i^- > 0\text{), and}$$
$$d^+ \geq 0 \text{ (some } d_i^+ > 0\text{)}.$$

From the Production Technology Requirement (1), a PU can move to a point (x',y') by either reducing inputs or by increasing outputs or by some combination of input reduction and output increase. The model in (6) is called a Mixed Technology Model because both inputs and outputs can be varied to find the comparison point. If we set the distance parameter $\xi^-=0$, the model is referred to as the Output Technology Model, meaning that outputs must be increased for an inefficient PU to move to the Efficiency Boundary. If we set the distance parameter $\xi^+=0$, the model is referred to as the Input Technology Model, meaning that inputs must be reduced to find the Efficiency Boundary.

The directions of descent and ascent can be used to impart special characteristics to the model. For example, when $d^+=y^j$ or $d^-=x^j$, the respective direction is referred to as the radial direction. It is shown in [10] that with certain other conditions, including the radial directions and the Input Technology Model, the model in (6) can be transformed into the Charnes, Cooper, and Rhodes (CCR) [5] model.

Another restriction on the direction allows one to model the situation of uncontrollable or nondiscretionary variables, meaning that a variable influences TE, but that the variable is not controllable by management. If the k-th input or output component is nondiscretionary, any direction other than $d_k^-=0$ or $d_k^+=0$ would represent

an unobtainable situation if the PU is inefficient since $\xi^- d_K^- > 0$ or $\xi^+ d_K^+ > 0$ means that the operating point must move some distance to locate a comparison point, but the component may not be changeable.

However, if a component direction is set to zero for a non-discretionary variable, then we must also assure that the component efficiency equals 1.0 for that variable. If there are r′ nondiscretionary input variables and r″ nondiscretionary output variables, we should exclude them from our TE calculation. However, the non-discretionary variables still have to be included in the model for finding the comparison point by setting the *direction to zero for* the nondiscretionary component. The next section will show how TE is determined when nondiscretionary variables are used.

2.5 Step 2: Determining the Value of TE.

The second step in measuring the TE of a PU is to find TE as some function of the comparison point, (x', y'). To illustrate how TE can be determined, we must first convert (3) to equation form by adding a non-negative slack variable, S^-. Similarly, we can make the inequality of (4) into an equality by adding a non-negative surplus variable, S^+. (3) and (4) are converted to equalities as follows:

$x_i^j - \xi^- d_i^- - S_i^- = x_i'$, where $\xi \geq 0$ is a scalar. Then, dividing by x_i^j, we have $1-(\xi^- d_i^- + S_i^-)/x_i^j = x_i'/x_i^j$. *From previous results,* $x_i'/x_i^j = \rho_i'$. $y_i' - S_i^+ = y_i^j + \xi^+ d_i^+$, and after dividing by y_i^j, we have $y_i'/y_i^j = \rho_i''$, and $\rho_i'' = 1 + (\xi^+ d_i^+ + S_i^+)/y_i^j$. Assuming that there are r′ nondiscretionary inputs and r″ nondiscretionary outputs with the corresponding directional component set to zero, the TE can be found as follows:

$$TE = \Big(\sum_{i \in K'} \rho_i'/(m-r') + \sum_{i \in K''} 1/(\rho''(s-r'')) \Big) \text{ for} \tag{7}$$

r′ nondiscretionary inputs and r″ nondiscretionary outputs,

$$K' = \{1,2,\ldots,m \mid x_i^j \text{ is discretionary}\},$$

$$K'' = \{1,2,\ldots,s \mid y_i^j \text{ is discretionary}\},$$

$$\rho_i' = 1-(\xi^- d_i^- + S_i^-)/x_i^j \quad \text{and} \quad \rho_i'' = 1+(\xi^+ d_i^+ + S_i^+)/y_i^j,$$

when ξ are obtained from (6). Note, if $d_i^- = 0$ and $S_i^- = 0$, then $\rho_i' = 1$. This condition corresponds to the previously defined condition of noncontrollable inputs.

2.6 The Generalized TE Model.

Thus far, we have treated the distance variables ξ^- and ξ^+ as equally important in locating the point on the input and output Efficiency Boundary. However, we may wish to allow for some other

priority to be given to either reducing inputs or increasing outputs in determining a comparison point. We use the weights $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ in the objective function of (6) to weight the importance of ξ^- and ξ^+, respectively.

Combining the two-step modeling considerations, including the use of nondiscretionary variables, weights, and directions, the General Mixed Input/Output Variable Efficiency-Improving Direction (VEID) Model becomes:

General Mixed Input/Output VEID Model

STEP 1: Find comparison point, (x', y') by solving:

$$\text{Max } \alpha_1 \xi^- + \alpha_2 \xi^+$$

$$\text{Subject To} \tag{8}$$

$$x^j - \xi^- d^- \geq z^T N = x', \quad d^- \geq 0 \text{ (some } d_i^- > 0)$$

$$y' = z^T M \geq y^j + \xi^+ d^+, \quad d^+ \geq 0 \text{ (some } d_i^+ > 0) \text{ and}$$

$$\xi^+, \xi^-, z \geq 0.$$

STEP 2: Find TE by calculating:

$$TE = \Big(wt_1 \sum_{i \in K'} \rho_i' + wt_2 \sum_{i \in K''} 1/\rho_i'' \Big), \text{ where}$$

$wt_1 = \alpha_1/(\alpha_1+\alpha_2)(1/(m-r'))$ if $m > r'$ and 0 otherwise,

$wt_2 = \alpha_2/(\alpha_1+\alpha_2)(1/(s-r''))$ if $s > r''$ and 0 otherwise,

$K' = \{1, 2, \ldots, m \mid x_i^j \text{ is discretionary}\}$

$K'' = \{1, 2, \ldots, s \mid y_i^j \text{ is discretionary}\}$

and there are r' nondiscretionary input and r'' nondiscretionary outputs with $d_k^- = 0.0$ and $d_k^+ = 0.0$, respectively,

where $\rho_i' = 1 - (\xi^- d_i^- + S_i^-)/x_i^j$ and $\rho_i'' = 1 + (\xi^+ d_i^+ + S_i^+)/y_i^j$ and $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $(\alpha_1 + \alpha_2) > 0$.

3. EXCESS COMPONENT CAPABILITY.

3.1 Introduction.

It has been shown that for an inefficient PU, the component efficiencies provide the information for determining the changes in inputs and outputs necessary for a PU to become efficient. When a PU is found to be efficient, however, each component efficiency is equal to 1.0 and the overall TE is 100%. Thus, if we have several PU with TE equal to 100%, and if we were to rank PU on the basis of TE, we could not discriminate either between the overall TE or the individual efficiency components. However, as will be illustrated using Figure 3, more information on PU which are TE can be determined after it is established that the PU is TE.

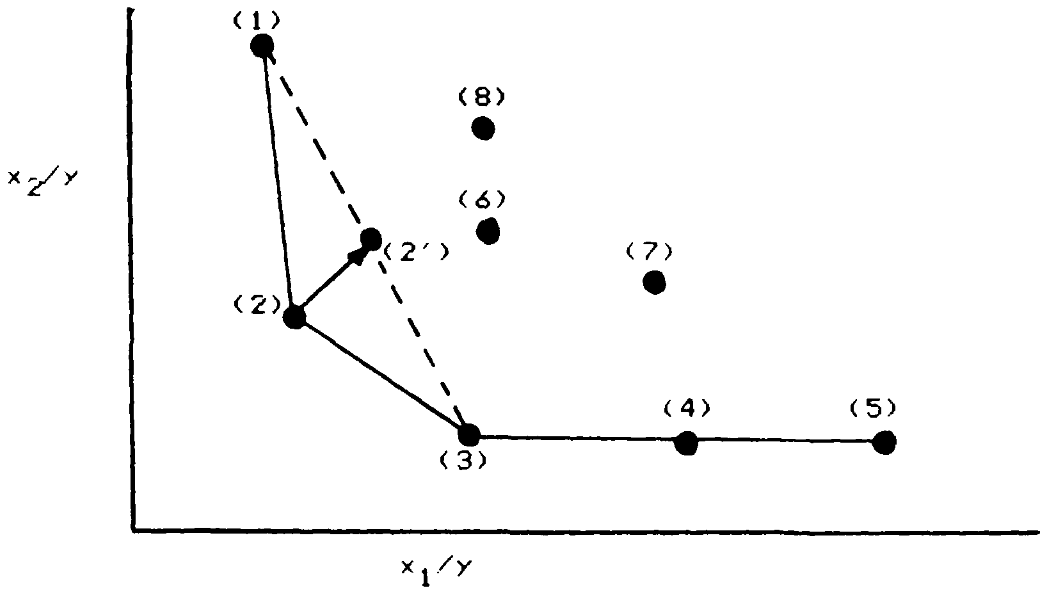

Figure 3. An Illustration of Excess Capability.

In Figure 3, consider PU (2), for example. If (2) were moved by increasing its inputs, the Efficiency Boundary would also move. As the inputs of (2) increase, eventually PU (2) would become inefficient. As PU (2) moves toward (2′), eventually the boundary will stop moving. Any movement beyond (2′) results in PU (2) moving into the interior of the Efficiency Boundary. Any further increase in inputs for (2′) will result in the TE for (2′) to be less than 100%.

Alternatively, as the output of PU (2) decreases, PU (2) moves toward the interior of the Efficiency Boundary. When all of the PU remain fixed and only PU (2) moves, the Efficiency Boundary will eventually become PUs (1), (3), (4), and (5). Eventually, any PU will become inefficient if moved in an interior direction a sufficient amount.

By moving an efficient PU to the point where it is no longer efficient and then finding a comparison point on the Efficiency Boundary, the value of the "excess capability" can be determined as the limit of the amount by which the PU can move and still remain TE. If the excess capability is zero, the PU is TE, but any change which takes the PU in an interior direction will cause the PU to become inefficient if all other PU remain at their operating point. For a PU which is already inefficient, the excess capability is negative, and any movement in an interior direction will cause the PU to become more inefficient. The method for finding the excess capability follows.

3.2 Method for Finding the Excess Capability Components.

Taking advantage of the symmetry of the problem gives us some insight into the determination of the Excess Production Capability.

First, three observations on the constraints of (8) can be made for a unit that is TE and for one that is not TE.

Condition	j-th PU is TE	j-th PU is not TE
1	$z_j=1$	$z_j=0$
2	$\xi^- d^-=0$	$\xi^- d^- \geq 0$
3	$\xi^+ d^+=0$	$\xi^+ d^+ \geq 0$

For the efficient PU, condition 1 means that the unit is its own comparison point, while the inefficient PU is never its own comparison unit. The comparison point of the inefficient unit is a composite of other efficient units. Condition 1 implies that the PU which is TE is on the boundary and the inefficient unit is not on the boundary. The inputs of the inefficient PU are above the boundary and the outputs are below the boundary, meaning that the inefficient PU must have its inputs decreased and its outputs increased to be efficient. From Figure 3, if we restrict $z_j=0$ for an efficient PU, the efficiency boundary moves up, meaning that the efficient PU inputs are now below the boundary. Similarly, the efficient PU outputs would be above the efficient boundary, meaning that outputs of the efficient PU could be decreased before the PU is not TE. The restriction $z_j=0$ also means that no PU is ever its own comparison unit. This restriction will "move" the boundary, but we have no prior knowledge of the efficiency of an arbitrary PU or if the direction to the input boundary is up (if the PU is TE) or down (if the PU is not TE).

By adding the restriction $z_j=0$, there is no change to the inefficient PU boundary since z_j will always be zero for PU that are inefficient. Accordingly, no changes to conditions 2 and 3 are needed for the inefficient PU because of this restriction. However, for the efficient PU, conditions 2 and 3 no longer hold if we use this added constraint. It will be shown now, however, that one more consideration will permit the model in (8) to determine both the efficient and inefficient PU and the excess production capability.

For condition 2, the input comparison point for the efficient PU will be above the j-th PU, so $\xi^- d^- \leq 0$. If we always select a direction with non-negative components, then $\xi^- \leq 0$. However, for the inefficient PU, if the direction components are non-negative, then $\xi^- \geq 0$. This implies that we can select any arbitrary non-negative direction for any PU and allow the distance value, ξ^-, to be unconstrained. If $\xi^- > 0$, the PU is not TE, but if $\xi^- < 0$, the PU is TE.

Similarly, condition 3 for the inefficient PU will remain valid, but must be changed for the efficient PU when the restriction $z_j=0$

is added. Again, by selecting non-negative ascent directions, then $\xi^+ d^+ \geq 0$, meaning $\xi^+ \geq 0$. For the efficient PU, the outputs are above the efficiency boundary, so $\xi^+ d^+ \leq 0$, meaning $\xi^+ \leq 0$ for the efficient PU. The output distance, ξ^+, when unconstrained, allows us to determine if the unit is TE by noting its sign.

One final consideration must be given for the Mixed Input/Output TE model. If a PU is efficient, both input and output distances must be less than or equal to zero. For the inefficient PU, both input and output distances must be greater than or equal to zero. These conditions are simultaneously met by requiring that $\xi^- \xi^+ \geq 0$. For example, if $\xi^- = 0$ and $\xi^+ > 0$, the unit is not TE. However, if $\xi^- < 0$ and $\xi^+ = 0$, the unit is TE. In both cases, the product of the distances is zero as required. The product constraint can be implemented procedurally as opposed to adding a nonlinear constraint. This will be discussed in section 3.8.

From the previous discussion, it can be seen that finding the comparison point for any PU can be made by a few modifications to the model in (8). The modifications which will permit us to find the excess production capability are:

<u>EXCESS PRODUCTION CAPABILITY MODEL CONSTRAINTS</u>

1. $z_j = 0$
2. ξ^-, ξ^+ unconstrained (9)
3. $\xi^- \xi^+ \geq 0$ (see section 3.8), and
4. $\rho_i' = \mathrm{Min}(1, x_i'/x_i^j)$, $\rho_i'' = \mathrm{Max}(1, y_i'/y_i^j)$.

3.3 Excess Production Capability Determined.

The modifications in (9) to the model in (8) evaluates the PU to determine if a PU is TE or not. However, the two-step process requires a determination of efficiency using the modified model (8) results. To determine the excess production capability and to show that the excess capability is the same as the inefficiency of a PU which is not TE, define the following:

$$\alpha_i' = x_i^* / x_i^j, \text{ where } x_i^* = x_i^j - \xi^- d_i^-$$

and (10)

$$\alpha_i'' = y_i^* / y_i^j, \text{ where } y_i^* = y_i^j + \xi^+ d_i^+.$$

The input and output component ratios for an efficient and inefficient PU obtained using (8) and (9) will be:

<u>Inefficient PU</u>	<u>Efficient PU</u>
$\alpha_i' \leq 1$	$\alpha_i' \geq 1$ (hence 4. in (9))
$\alpha_i'' \geq 1$	$\alpha_i'' \leq 1$ (hence 4. in (9))

The conventional definition of efficiency restricts input

components to be less than or equal to 1.0. Any component less than 1.0 means that the component is inefficient. Similarly, then, any value greater than 1.0 must represent an excess production capability. Therefore, instead of defining input and output component efficiencies, the excess capability will be found relative to the value 1.0. With the component ratios in (10), a value greater than 1.0 could be found, so define the Input Excess component capability as:

$$IE_i = \alpha_i' - 1. \tag{11}$$

If $\alpha_i'<1$, the excess component capability is the measure of inefficiency. If $\alpha_i'>1$, the value of IE_i is the amount by which the *i*-th component can be increased before the unit is not TE and, therefore, represents the excess capability.

The Output Excess component capability is similarly defined:

$$OE_i = 1/\alpha_i'' - 1. \tag{12}$$

If $\alpha_i''<1$, the unit is TE, so $1/\alpha_i''>1$ and $OE_i>0$, meaning that the i-th output component can be reduced and the unit will remain TE. However, if $OE_i<0$, the value represents the measure of the PU's inefficiency, meaning the output component has to be increased to make the PU efficient.

It can be seen that IE_i and OE_i both measure the change in the components relative to a boundary which excludes the j-th PU as a comparison point. Values greater than zero represent the excess component capability, while values less than zero are the component inefficiency. Both are measured relative to the theoretical value 1.0, the value a component efficiency would take if the PU were TE.

The comparison point, $(x'',y'')=(x^j-\xi^-d^-,y^j+\xi^+d^+)$ may not be an efficient point. From the previous discussion, the point $(x',y')=(z^TN,z^TM)$ is an efficient comparison point. The difference between x' and x'' is the slack which converts the inequality in (3) to an equality. The difference between y' and y'' is the surplus which converts the inequality in (4) to an equality. Another definition of excess capability using an efficient comparison point can be made as explained below.

The efficient input and output component ratios similar to (10) are defined as follows:

$$\lambda_i' = x_i'/x_i^j,\ \text{where } x_i'=(z^TN)_i$$

and (13)

$$\lambda_i'' = y_i'/y_i^j,\ \text{where } y_i'=(z^TM)_i.$$

Similar to (11), the Input Excess Efficient component capability is defined as

$$IEE_i = \lambda_i' - 1 \text{ and} \tag{14}$$

the Output Excess Efficient component capability is defined similar to (12) as

$$OEE_i = 1/\lambda_i'' - 1. \tag{15}$$

Whereas the Input and Output Excess component capabilities (IE_i and OE_i) show the amount by which an efficient PU can move and remain TE, the Input and Output Efficient Excess component capabilities (IEE_i and OEE_i) show the change in efficiency if the efficient PU in- and outputs are moved beyond the boundary. If a PU is not TE, the values of the Excess Efficient component capabilities are the measures of inefficiency relative to an efficient point on the boundary.

3.4 The Input Excess Capability.

The Input Excess Capability and the Input Excess Efficient Capability components must be computed to form the two measures of Input Excess Capability. Using (8) and (9) with suitable interior descent and ascent directions, the values for (10) and (11) are calculated for each input component.

The Average Input Excess Efficient capability is:

$$AEE' = \sum_{i=1}^{m} IEE_i/m. \tag{16}$$

Total Efficiency with Excess Input Efficient Capability is:

$$TEEC' = (100 + AEE')\%.$$

The Average Input Excess capability is:

$$AE' = \sum_{i=1}^{m} IE_i/m. \tag{17}$$

The Total Efficiency with Excess Input Capability is:

$$TEC' = (100 + AE')\%. \tag{18}$$

Note that TEEC′ and TEC′ may not be equal. Section 3.7 will illustrate the two Input Excess capabilities and show that the Average Excess Capabilities may be less than 100%, meaning that the PU is not TE or is an extreme point.

3.5 The Output Excess Capability.

Similar to section 3.4, two excess output capabilities for the j-th PU can be determined.

The Average Output Excess Efficient capability is:

$$AEE^{*} = \sum_{i=1}^{s} OEE_i/s. \tag{19}$$

Total Efficiency with Excess Input Efficient Capability is:

$$TEEC^{*}=(100 + AEE^{*})\%. \tag{20}$$

The Average Output Excess capability is:

$$AE^{*} = \sum_{i=1}^{s} OE_i/s. \tag{21}$$

The Total Efficiency with Excess Output Capability is:

$$TEC^{*}=(100 + AE^{*})\%. \tag{22}$$

3.6 Illustration of TE Measurement.

In order to give a more general illustration of the Efficiency Boundary and how an efficient PU's excess capability is found, consider the following data on ten PU, each of which have two inputs and two outputs. The TE and other data has also been given for the ten PU using the VEID model.

TABLE 1: TE of Ten Producing Units.

j=PU Number	Inputs x^j	Outputs y^j	TE Input[1] Technology	TE Output[2] Technology	TE Mixed[3] Technology
1	7,4	10,10	100.0	100.0	100.0
2	5,6	8,12	100.0	100.0	100.0
3	4,10	6,15	100.0	100.0	100.0
4	8,8	12,12	97.1	84.4	92.2
5	9,6	10,10	76.4	73.5	86.7
6	12,3	12,8	100.0	100.0	100.0
7	18,3	14,6	100.0	100.0	100.0
8	15,5	10,8	61.7	60.9	80.4
9	13,7	12,12	68.4	67.5	83.8
10	11,9	13,13	79.0	72.2	86.1
Average	10.2,6.1	10.7,10.6	89.4	85.8	92.9

Notes:

1. Radial descent direction.
2. Radial ascent direction.
3. Equal weights for the mixed model was used with the directions in notes 1 and 2.

From Table 1, it can be seen that for every PU which is TE,

the unit has TE=100% no matter which model is used. However, for a PU which is not TE, the type of model used may cause the TE to vary for the same PU. This would also be true if different ascent or descent directions are used.

3.7 Illustration of Input Excess Capability.

Because Figures 2 and 3 used the special case of a single output, the efficiency boundary of an inefficient PU with multiple outputs is not clear. Figure 4 shows the input efficiency boundary for PU (8). Note that the inputs of PU (8) are inside the efficiency boundary. This means that the excess efficient input capability is negative and is equal to the value of the inefficiency of PU (8). The line segments denoted as SLACK are comparison points, but are not efficient comparison points. The efficient comparison points are found at the intersection of A0 and A′0 and the SLACK segment. Along the slack line, AE′>AEE′ and AE′=AEE′ on all other points on the efficiency boundary.

By varying the direction for finding an inefficient interior point and using the method in section 3.3, the Efficiency Boundary for the efficient PU in Table 1 can be drawn. Figures 5 through 9 show the Efficiency Boundary for PU (1), (2), (3), (6), and (7). In illustrating the boundary, several directions may be shown which are not interior directions.

In Figures 5 through 9, the cone of all non-negative linear combinations of PU used as comparison points is denoted as A0A′.

Consider first Figure 5. For PU (1), the operating point lies below the efficiency boundary. Several directions are shown by lines with arrows which point toward the interior of the efficiency boundary. From Figure 5, it can be seen that the distance from (1) to the efficiency boundary differs, depending upon the direction taken toward the interior of the boundary. Therefore, the input excess capability will also vary, depending upon the direction taken. Any point along the line segments marked SLACK will have AEC′>0, but AEEC′<AEC′ (with AEEC′<0 in some instances) since the efficient comparison points are on the intersection of A0 or A′0 and the SLACK segments.

Now consider Figure 6, which shows the Efficiency Boundary for PU (2). Notice the inclusion of the vertical line labeled SLACK. If the vertical direction is taken, the input component x_2 can be increased indefinately and PU (2) will always be efficient. However, when a direction is taken which also has a positive horizontal compone , an inefficient point will be located. When the direction intersects the line called SLACK, the efficient comparison point is located by moving toward the origin along the ray A0 or A′0. The distance between where the direction intersects the SLACK and the

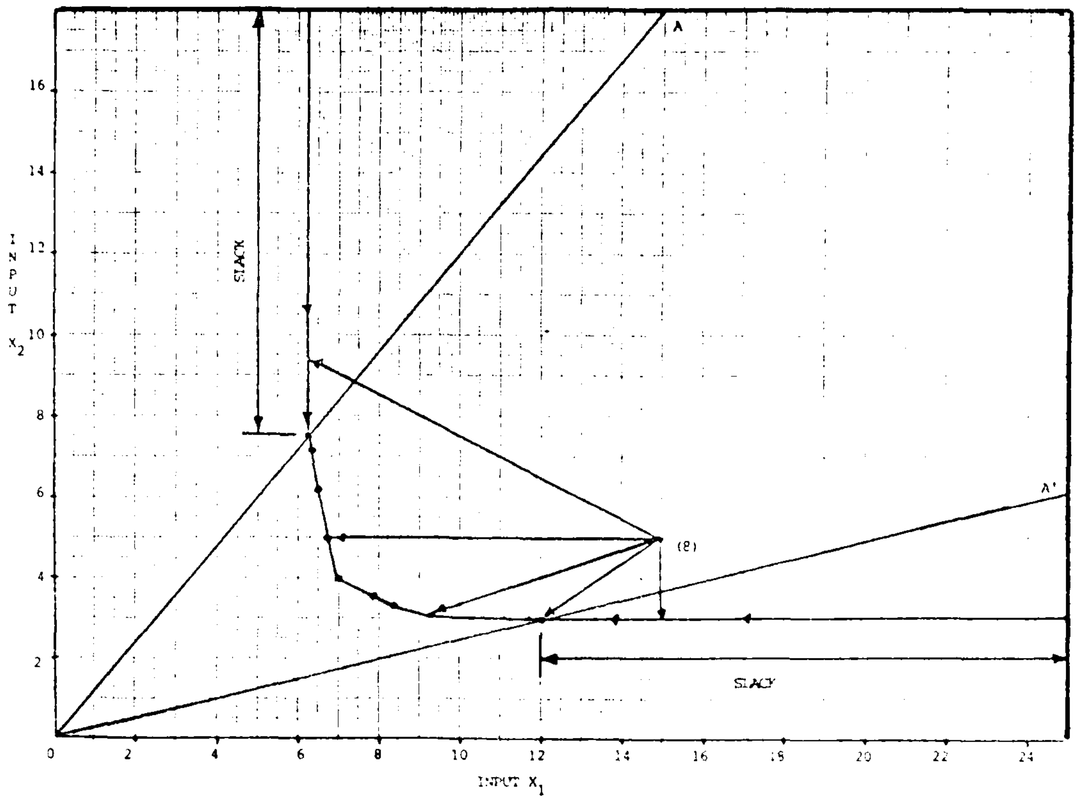

Figure 4. PU 8 EFFICIENCY BOUNDARY

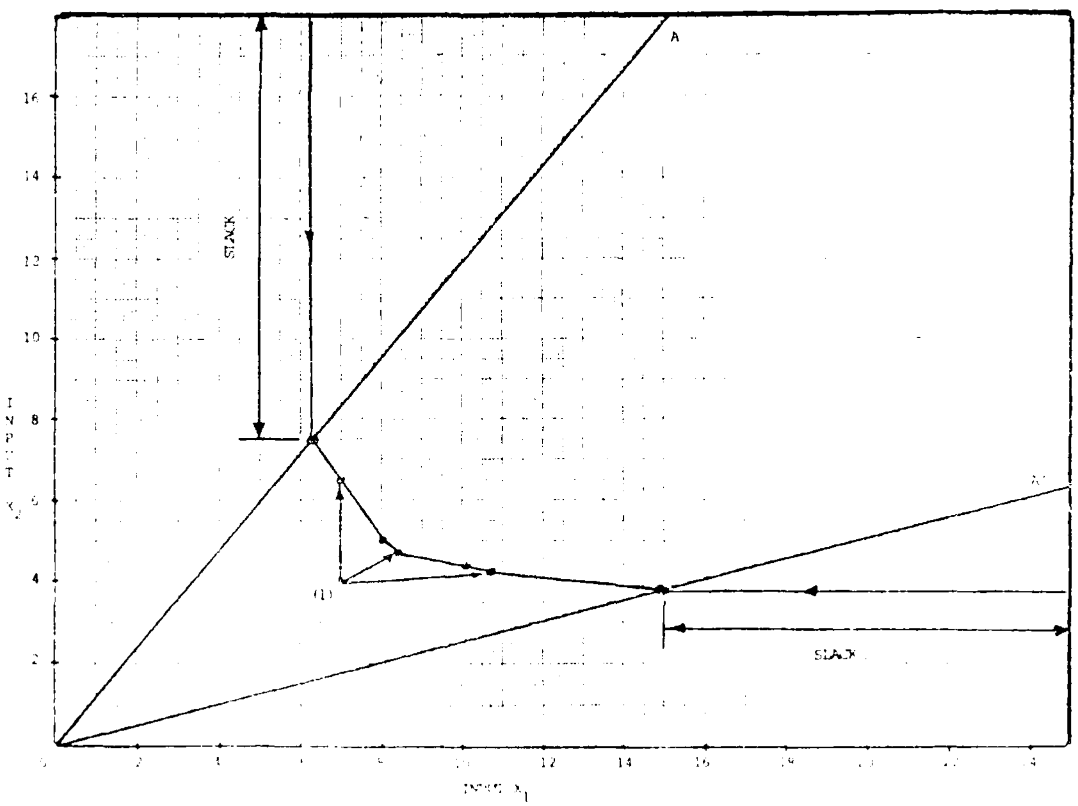

Figure 5. PU 1 EFFICIENCY BOUNDARY

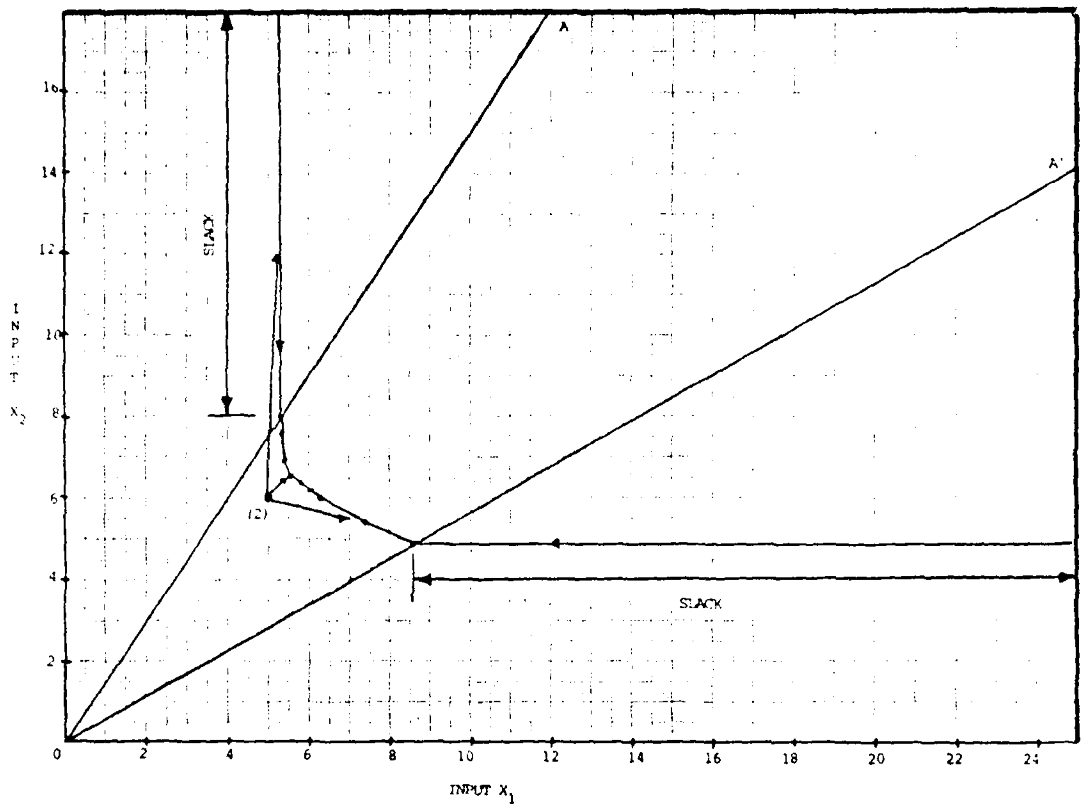

Figure 6. PU 2 EFFICIENCY BOUNDARY

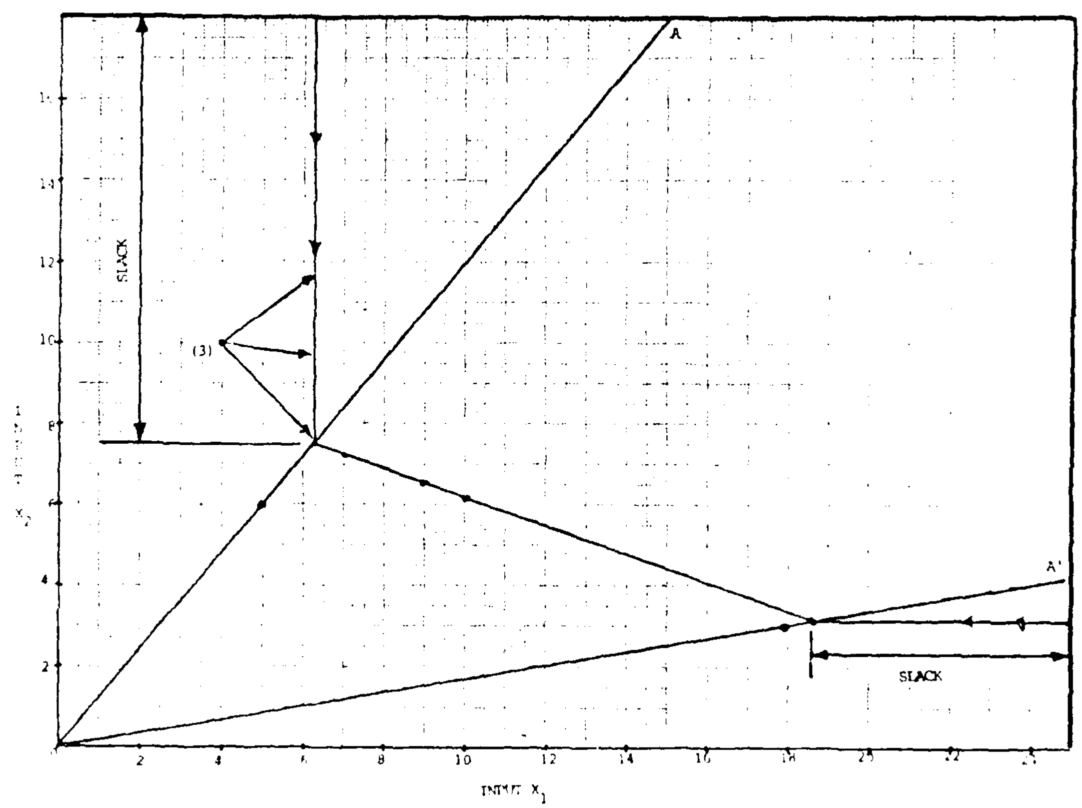

Figure 7. PU 3 EFFICIENCY BOUNDARY

efficient comparison point is the amount of slack in the mathematical program for the constraint on the operating point, x_2^2.

Figure 7 shows the Efficiency Boundary for PU ([illegible]). Again, the line marked SLACK is indicated. When the direction intersects the SLACK line, an efficient comparison point is found by going down the SLACK line until it intersects the rays of the cone AOA′. The solution of the input constraints will have a non-zero slack variable in the second component constraint. Notice that if PU (3) is moved until it is inefficient, then all of the non-negative linear combinations of efficient PU will exclude PU (3). Therefore, the cone AOA′ will be determined by PU (2) and PU (7) as shown. A vertical or horizontal direction will not intercept either the slack line or the efficiency boundary. Therefore, an inefficient point cannot be found following these directions. Note that neither of these are interior directions.

Figure 8 shows the Efficiency Boundary for PU (6). PU (6) can be increased indefinitely in the horizontal direction and will never become inefficient.

Figure 9 shows the Efficiency Boundary for PU (7). Because of the horizontal slack line, PU (7) can be increased indefinitely in the x_1 (horizontal) direction and will remain efficient.

For points on the efficiency boundary with slack values, if AEE′ were calculated, we may find that AEE′<0. This situation will arise because, after both inputs are increased, eventually PU (7) becomes inefficient, and to become efficient again, the PU has to decrease its x_1 component by the amount of the slack. When PU (7) becomes inefficient, the cone of Feasible Points, AOA′, is determined by PU (6). This means that once its inputs are increased, PU (7) has to "shed" a large part of its first input component to become efficient again.

To summarize figures 4 through 9, PU (2) and (3) can be increased indefinitely in the vertical direction and PU (6) and (7) can be increased indefinitely in the horizontal direction. PU (1) will become inefficient in either horizontal or vertical direction. This situation arises because, as the direction approaches the vertical direction for PU (2) and (3) or the horizontal direction for PU (6) and (7), the slack value increases. With the direction where an input component can increase infinitely and the PU remains TE, the slacks would have to be infinitely large. It can be shown that an interior direction cannot result in an infinite increase in inputs before the PU becomes inefficient.

Table 2 illustrates the results of using the Input and the Output Excess Capability Methods on the set of input and output data in Table 1.

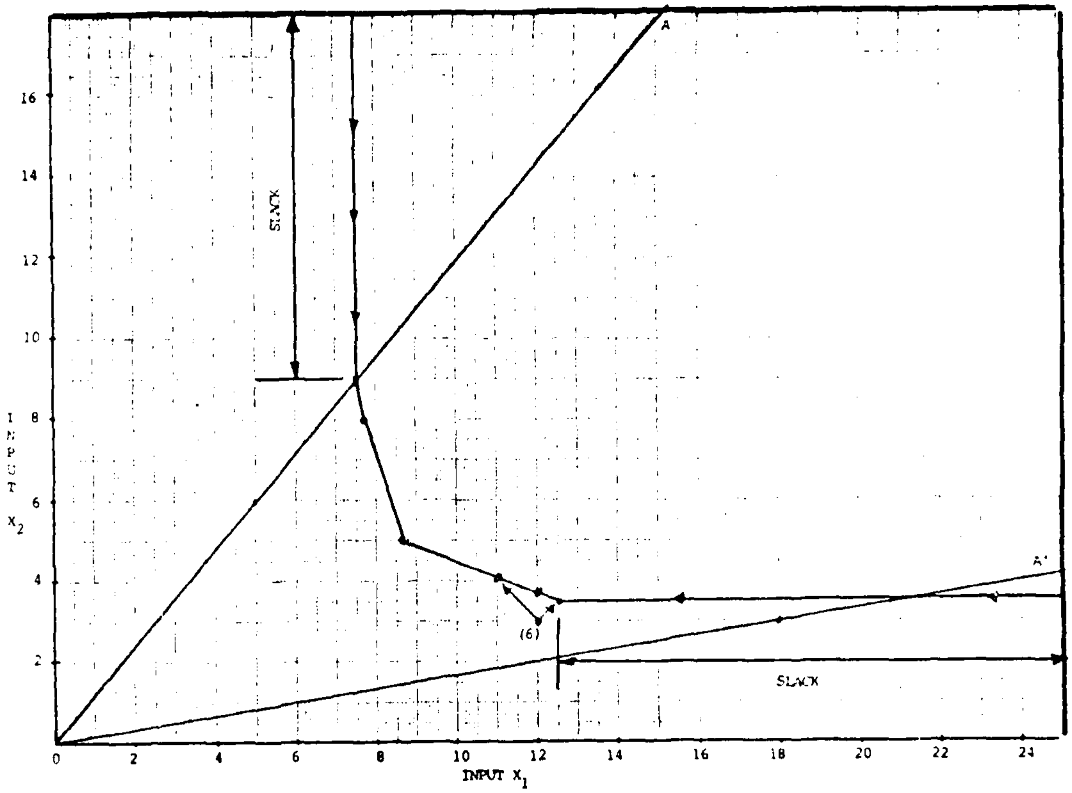

Figure 8. PU 6 EFFICIENCY BOUNDARY

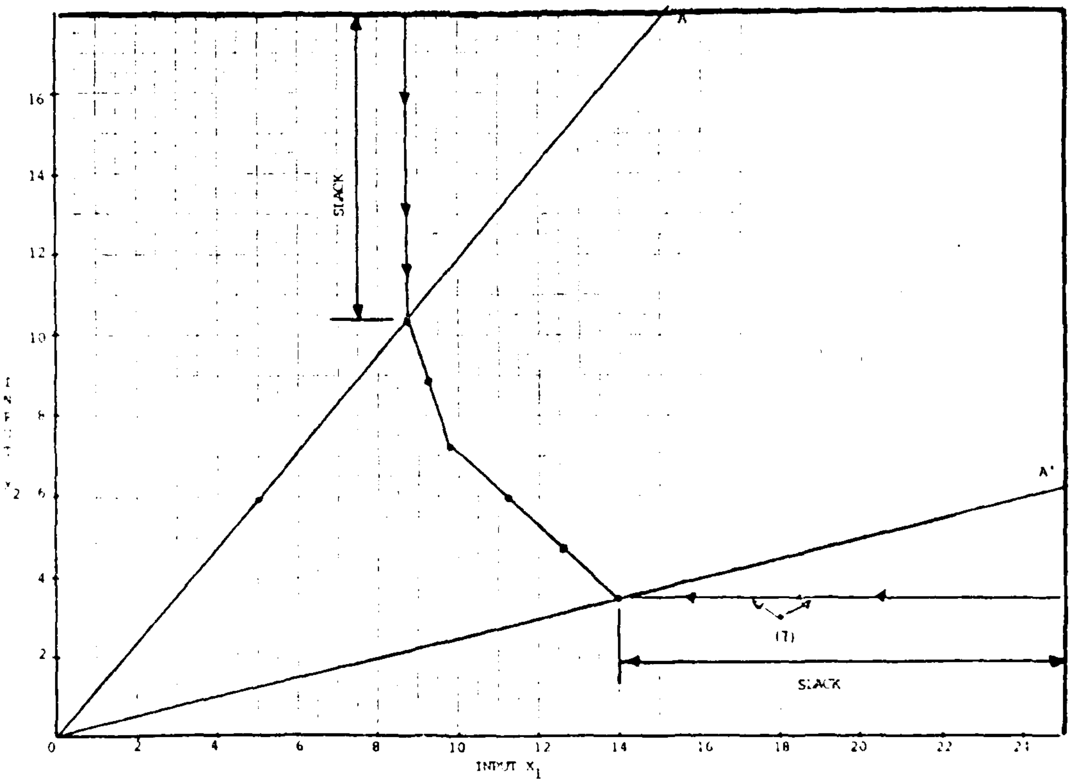

Figure 9. PU 7 EFFICIENCY BOUNDARY

Table 2: Excess Capability.

j=PU	Input Excess Capability $TE^{1,3}$	AE'	AEE'	Output Excess Capability $TE^{2,4}$	AE''	AEE''
1	100.0	18.5	18.5	100.0	18.4	18.4
2	100.0	10.1	10.1	100.0	12.8	10.0
3	100.0	56.3	15.6	100.0	524.5	25.0
4	97.1	- 2.9	- 2.9	84.4	- 2.9	-15.6
5	76.4	-23.6	-23.6	73.5	-23.5	-26.6
6	100.0	16.7	10.4	100.0	15.1	15.1
7	100.0	16.7	- 2.8	100.0	33.0	- 4.2
8	61.7	-38.3	-38.3	60.9	-35.8	-39.1
9	68.4	-31.7	-31.7	67.5	-31.8	-32.5
10	79.0	-21.0	-21.0	72.2	-20.9	-27.9

Notes:

1. Input Technology Model.
2. Output Technology Model.
3. Directions are radial.
4. $d^{-}=(\bar{x}_1,\bar{x}_2)$ for inputs and $d^{+}=(\bar{y}_1,\bar{y}_2)$ for outputs.

From Table 2, using the radial direction, inputs can be increased an average of 18.5% and PU (1) will remain TE. It can be shown for the radial case that the AE′=AE". For PU (1), outputs can be decreased an average of 18.4% using the given direction and PU (1) will remain TE. Also, PU (3) has the larger excess capability for the directions used in Table 2. However, as seen in Figure 7, the large excess capability for PU (3) is because PU (3) is an extreme point with no close efficient PU. The large AE′ or AE" with a much smaller AEE′ or AEE" is an indication of specialization in a single input or output. When no longer efficient, the excess input must be shed or the low output component must be increased to bring the specializing PU into line with the other efficient PU.

The symmetry of the approach presented is now complete. Note that the inefficiency is a measure of the average change which a PU must make to become efficient, while the excess production capability is the measure of the amount by which an efficient PU can change before it becomes inefficient. A negative excess capability is the same as inefficiency. A negative value of inefficiency is the same as a positive excess capability. When PU are efficient, the inputs can be increased, so the PU is below the input efficiency boundary. When a PU is inefficient, the PU must decrease its outputs, so it is below the efficiency boundary. The values of an efficient input excess component capability are greater than or equal to 1.0 as are

the excess component capabilities of an inefficient PU output. The list of the similarities goes on.

3.8 Product Constraint and Unconstrained Variables.

A typical way of handling unconstrained variables in a Linear Program (LP) is to change the unconstrained variable into the difference of two non-negative variables. Using this technique, then

$$\begin{aligned} \xi^- &= \xi_1^- - \xi_2^-, \text{ where } \xi^- \text{ is unconstrained and} \\ &\qquad \xi_1^-, \xi_2^- \geq 0, \\ \xi^+ &= \xi_1^+ - \xi_2^+, \text{ where } \xi^+ \text{ is unconstrained and} \\ &\qquad \xi_1^+, \xi_2^+ \geq 0. \end{aligned} \tag{23}$$

The methodology which solves the LP has a rule for replacing one positive variable with another, depending upon which variable can improve the objective function. The product restriction $\xi^+\xi^- \geq 0$ can be implemented procedurally once a feasible solution is obtained by restricting the entry of ξ_2^+ if $\xi_1^- > 0$ and restrict ξ_1^+ if $\xi_2^- > 0$.

3.9 Excess Production Capability: A Measurement of Overproduction.

By the symmetry argument, if inefficiency represents the amount of improvement needed for a PU to become TE, then the positive excess production capability represents the amount by which an efficient PU's performance can be degraded before the PU becomes inefficient. Therefore, the positive output excess efficient capability component is a measurement of the amount of production which a PU has produced over and above that which is necessary to be TE, or the amount of overproduction. In determining the efficient comparison point, y', for an inefficient PU, the output will be adjusted upward to determine what <u>should</u> be produced. Similarly, then, for the efficient PU, the output will be adjusted downward to determine y', the amount which the efficient PU <u>should</u> produce to remain TE. The next chapter will use the efficiency adjustments to determine the production effectiveness of PU.

4. <u>EFFECTIVENESS.</u>

4.1 Component Effectiveness Defined.

The ratio of actual production to mission is a typically used measure of mission effectiveness. However, from the previous discussion, the actual production may not be that which is achievable by the PU. Also, in the case where PU have multiple outputs, as in the measures of efficiency, a component measurement of effectiveness will be defined. Using (8), the output production which could be expected if a producing unit were technically efficient can be found. The

Actual and Adjusted Mission Effectiveness are defined as follows:

$$\tau_{ij} = y_i^j / M_{ij}$$

and (24)

$$\tau'_{ij} = y'_i / M_{ij}, \text{ for the i-th output of the j-th PU, and}$$

τ_{ij} is the Actual Mission Effectiveness Ratio,

τ'_{ij} is the Efficiency Adjusted Mission Effectiveness Ratio,

y_i^j is the Actual Production,

y'_i is the Efficiency Adjusted Production,

and M_{ij} is the j-th PU's Mission for the i-th Output.

If $y'_i > M_{ij}$, then $\tau'_i > 1$, and $\tau'_i > 0$ since positive production output and mission is assumed. Also, if $y'_i < M_{ij}$, then $\tau'_i < 1$. By calculating the value $\beta'_{ij} = 1 - \tau'_{ij}$ or $\beta_{ij} = 1 - \tau_{ij}$, if:

$\beta'_{ij} = 1 - \tau'_{ij} > 0$, the j-th PU is overmissioned, and if
$\beta'_{ij} = 1 - \tau'_{ij} = 0$, the j-th PU is properly missioned, and if
$\beta'_{ij} = 1 - \tau'_{ij} < 0$, the j-th PU is undermissioned for the i-th

output, with respect to what the PU <u>should</u> produce. A similar relationship holds for β_{ij}, which is a measurement of the Actual Mission Effectiveness. Overmissioning implies underproduction and undermissioning implies overproduction.

In a large scale operating environment, it would be expected to find many cases where $\beta_{ij} \neq 0$. However, we would not expect to find too many instances of large overmissioning or undermissioning. Put another way, we should expect the distribution of adjusted production output to be statistically the same as the distribution of mission for individual outputs as well as for outputs combined. The following method allows one to check the "goodness of fit" between the mission and adjusted production output.

An appropriate statistic for comparing two distributions is the Chi-square statistic, which is calculated as follows:

$$x^2 = \sum_{j=1}^{n} \frac{(f_e^j - f_o^j)^2}{f_e^j}, \text{ where} \qquad (26)$$

f_e^j is the j-th "expected" frequency,

f_o^j is the j-th "observed" frequency, and

there are n "cells" being compared.

The expected frequencies in the production environment are the

missions assigned to the PU and the observed frequencies are the j-th adjusted production or actual production. (26) can be used to measure the fit of mission to the actual production or to the adjusted production of all of the PU. The n "cells" are the n PU. To determine if the actual Mission and Adjusted (or Actual) production "fit", a critical value of the Chi-squre statistic is determined at some level of significance, α, and n-1 degrees of freedom. If the value of (26) exceeds the critical value, it is concluded that the production does not fit the mission. In this case, the values of β'_{ij} show the over- and undermissioning of the PU.

The chi-square test statistic can be found in terms of the Production Effectiveness Components, β'_{ij} values as shown below:

$$\chi^2 = \sum_{j=1}^{n} (\sqrt{M_{ij}}\,\beta'_{ij})^2 = \sum_{j=1}^{n} M_{ij}(1-y'_i/M_{ij})^2 \qquad (27)$$

$$= \sum_{j=1}^{n} (M_{ij} - y'_i)^2/M_{ij} = \chi^2.$$

Chi-squre values can be calculated for each individual output separately or for combinations of outputs. Also, the Chi-squre test can be used to test the fit of mission to Actual Production as well as to Adjusted Production as in (27). If (27) is used for more than one output, the degrees of freedom for the critical value is one less than the number of total terms being summed over.

It may also be of interest to calculate the Chi-square statistic for the efficient and inefficient PU separately to see how the mission process affects production. Also, if several periods of time (say quarters) are used simultaneously, Chi-square values for each individual time and the Chi-square over all four quarters can be determined using (27).

If the Chi-square statistic is larger than the critical value, the implication is that there are significant under- and overmission values. The following method allows us to determine those PU which either have significantly large or small Efficiency Adjusted Efficiency Components (β'_{ij}). Using the critical value for a level of significance, α, then we will accept the hypothesis that mission fits Adjusted (or Actual) Production if:

$$\chi^2_{test} \le \chi^2_{critical}, \text{ or} \qquad (28)$$

$$\sum_{j=1}^{n} \frac{(f_o^j - f_e^j)^2}{f_e^j} \le \chi^2_{critical}. \qquad (29)$$

If every term of (29) is less than 1/n times the Chi-square critical value, then (29) will be satisfied, so let

$$(f_o^j - f_e^j)^2/f_e^j \le \chi^2_{critical}/n. \tag{30}$$

From (30), this means that

$$M_{ij}\beta^2_{ij} \le \chi^2_{critical}/n, \text{ or} \tag{31}$$

$$|\beta_{ij}| \le \chi_{critical}/\sqrt{nM_{ij}}. \tag{32}$$

The expression in (32) allows us to identify those component effectiveness values which, even if the Chi-squre indicates no significant difference, are excessive. The expression also holds for the Efficiency Adjusted Component Effectiveness values.

4.2 Illustration of Effectiveness Measurement.

Consider the following example, which shows the two inputs and two outputs for the same ten PU in Table 1. Table 3 also shows the ten PU's respective mission.

Table 3. Example of Adjusted Mission Effectiveness.

	1	2	3	4	5	6	7	8	9	10
Inputs	7,4	5,6	4,10	8,8	9,6	12,3	18,3	15,5	13,7	11,9
Outputs	10,10	8,12	6,15	12,12	10,10	12,8	14,6	10,8	12,12	13,13
Efficiency	100.	100.	100.	84.4	73.5	100.	100.	60.9	67.5	72.2
y'_1	8.4	7.3	6.4	12.4	13.1	10.7	12.0	16.7	18.0	16.5
y'_2	8.5	10.9	9.6	16.7	14.2	6.8	8.0	13.0	17.6	19.9
M_{1j}	9.0	7.5	7.5	12.0	9.5	11.0	14.0	9.0	11.0	12.0
M_{2j}	10.5	11.0	14.0	12.0	8.0	7.5	7.0	8.0	13.0	14.0
$1-\tau_{1j}$	-11.0	-6.7	20.0	0.0	-5.3	-9.1	0.0	-11.1	-9.1	-8.3
$1-\tau_{2j}$	4.8	-9.1	-7.1	0.0	-25.0	-6.7	14.3	0.0	7.7	7.1
$1-\tau'_{1j}$	6.3	3.2	14.7	-3.0	-37.8	2.4	14.3	-85*	-63*	-37.1
$1-\tau'_{2j}$	19.5	0.7	31.4	-39*	-77*	10.0	-14.3	-62*	-35.0	-42*

Notes:

1) * values are considered excessive using (32).
2) y' for each PU is found by (8) with the three conditions for calculating excess production capability with the output technology model and ascent direction $d^+=(\bar{y}_1,\bar{y}_2)=(10.7,10.6)$.
3) Component Effectiveness $(1-\tau_{ij})$ found using (24) are in percentages.
4) Values have been rounded.

Several observations from Table 3 can be made. First, PU 4, for

example, produced exactly what is was missioned to produce. Because it is inefficient, however, PU 4 is actually undermissioned 3% and 39% for Output 1 and 2, respectively. Also, if we were to compare PU 7 production with mission, we would conclude incorrectly that PU 7 is overmissioned by 14.3% for Output 2. However, when compared to what it should produce, PU 7 is actually overmissioned by 14.3% on Output 1 and undermissioned by 14.3% on Output 2. These examples illustrate that the level of resources influences the production which should be achieved by individual PU and, therefore, should affect its mission. Only when Mission equals what a PU should be able to produce will the Actual Production Effectiveness be a valid indicator of the successful PU. For this example, the proper level of mission for the inefficient PU should have been the y' values and the efficient PU which exceeded the y' values should receive the recognition for overproducing.

From Table 3, it can also be seen that there are two values in the Efficiency Adjusted Component Effectiveness for Output 1 which, by (32), are considered to be excessive. Also, there are four values in the Efficiency Adjusted Component Effectiveness for Output 2 ($1-\tau_{2j}$) which are considered to be excessive. Table 4 shows which of the Chi-square tests are significant and sheds more light onto the problem of evaluating the Effectiveness.

Table 4. Chi-square Test Statistics.

	ACTUAL OUTPUT 1	ACTUAL OUTPUT 2	ACTUAL BOTH 1&2	ADJUSTED OUTPUT 1	ADJUSTED OUTPUT 2	ADJUSTED BOTH 1&2
AVG β_{ij}	-4.1%	-1.4%		-18.6%	-20.8%	
χ^2	0.85	1.01	1.86	14.50**	15.81**	30.31**
χ^2_{TE}	0.54	0.36	0.90	0.50	2.00	2.50
χ^2_{NTE}	0.31	0.65	0.96	14.01*	13.81*	27.82**

Notes:

1) * values are significant at the .05 level.
2) ** values are significant at the .10 level.
3) Avg β_{ij} values are the average values of $1-\tau_{1j}$, $1-\tau_{2j}$, $1-\tau'_{1j}$, and $1-\tau'_{2j}$ values in Table 3, respectively.
4) Subscripts TE and NTE for Chi-square are calculated for the Technically Efficient and Not Technically Efficient PU, respectively.

From Table 4. several observations can be made. First, looking at the Actual Mission Effectiveness, we would conclude that the

actual production fits the mission. This is seen by noting that the AVG β_{ij} values for both outputs are small (-4.07% and -1.40%) and the Chi-square values using the actual production are small.

However, when the Efficiency Adjusted Production values are used to determine production effectiveness, it can be seen that Output 1 is actually being undermissioned by an average of 18.6% and Output 2 is actually being undermissioned by an average of 20.8% The Chi-square test indicates a significant difference between the mission and production for all PU, with the greatest difference being in the NTE (Not Technically Efficient) PU. From this Table, we would conclude that the major problem with the PU in this example is that inefficient PU are undermissioned with respect to what they should produce, but actually are properly missioned for what they actually produce. Thus, we might also conclude that the missioning process may, indeed, be a major contributing factor in the inefficiency of the PU if the PU are producing only the amount for which they have been missioned.

5. CONCLUSIONS.

Two methods for finding the excess capability of efficient PU have been presented for inputs and for outputs. By using these two algorithms, a new efficiency can be found which includes the average input increase or output decrease which an efficient PU can have and still remain TE. The excess production capability of the efficient PU is a measurement of the overproduction of the PU. In determining the excess production capability of either the efficient or inefficient PU, an adjustment can be made to the actual production to determine what the PU should produce. The determination of what a PU should produce allows us to determine the Production Effectiveness and, therefore, how well the mission fits actual or efficiency adjusted production.

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Efficiency and Effectiveness are two measures commonly used to evaluate Production Units (PU). Whereas the Efficiency measurement shows how well a PU transforms its inputs into outputs and prescribes a corrective course of action to make an inefficient PU efficient relative to other similar PU, the Effectiveness measurement shows how well a PU is producing relative to a management defined mission or goal for production output. This report shows how Efficiency affects Effectiveness and presents a methodology for statistically comparing a PU's production to mission.

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ABSTRACT (Continued)
A method for ranking both efficient and inefficient PU according to their "excess production capability" is introduced. This methodology also permits one to determine the overproduction of a PU, which is essential in the Effectiveness measurements. The material in this report will be applied to a subsequent separate report to the fifty-six US Army Recruiting Battalions.

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